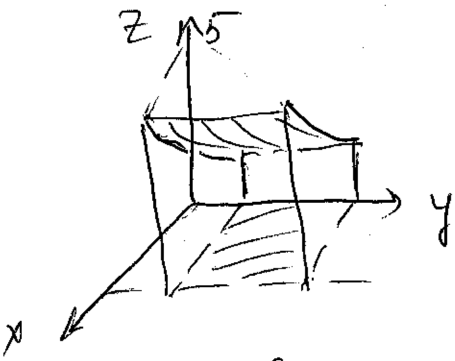
חדו"א להנדסת מכונות 2

דף 9. פתרון

① א. $I=\iiint_T y\,dx\,dy\,dz$

$T: \quad 0\le x\le 2$
$\quad\quad 1\le y\le 3$
$\quad\quad 0\le z\le x^2+1$



$$I=\iint_D dx\,dy\int_0^{x^2+1} y\,dz=$$

$$=\int_0^2 dx\int_1^3 y\,dy\int_0^{x^2+1} dz=$$

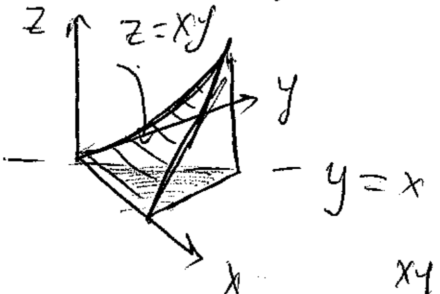
$$=\int_0^2 dx\int_1^3 (x^2+1)\cdot y\,dy=\int_0^2 (x^2+1)\cdot \frac{y^2}{2}\Big|_{y=1}^{y=3}dx=$$

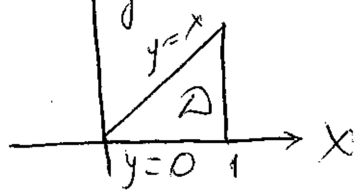
$$=\int_0^2 4(x^2+1)\,dx=4\cdot\left(\frac{x^3}{3}+x\right)\Big|_0^2=4\cdot\left(\frac{8}{3}+2\right)=$$

$$=\frac{56}{3}$$

ב. $I=\iiint_T x y^2 z^3\,dx\,dy\,dz$

T חסום ע"י המשטחים: $z=xy$, $z=0$, $y=x$, $x=1$





$$I=\iint_D dx\,dy\int_0^{xy} xy^2z^3\,dz=\iint_D xy^2\cdot\frac{z^4}{4}\Big|_{z=0}^{z=xy}dx\,dy=$$

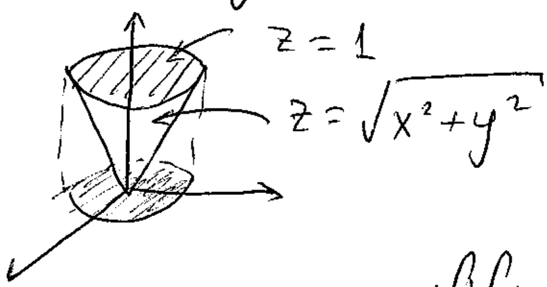
$$= \frac{1}{4}\int_0^1 dx \int_0^x x^5 \cdot y^6\, dy = \frac{1}{4}\int_0^1 x^5 \cdot \frac{y^7}{7}\Big|_{y=0}^{y=x} dx =$$

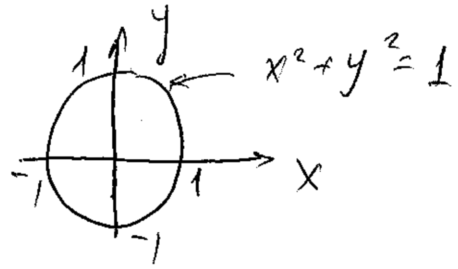
$$= \frac{1}{4}\int_0^1 x^5 \cdot \frac{x^7}{7}\, dx = \frac{1}{28}\cdot\int_0^1 x^{12} dx = \frac{1}{28}\cdot\frac{x^{13}}{13}\Big|_0^1 =$$

$$= \frac{1}{28\cdot 13} = \frac{1}{364}$$

$$I = \iiint_T \sqrt{x^2+y^2}\, dx\,dy\,dz \quad \text{.א} \qquad (2)$$

T חסום ע"י המשטחים $z^2=x^2+y^2$, $z=1$





נעבור לקואורדינטות גליליות!

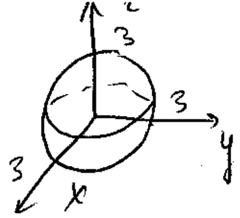
$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \qquad \begin{aligned} dx\,dy\,dz &= r\,d\theta\,dr\,dz \\ 0 \le \theta &\le 2\pi \\ 0 \le r &\le 1 \\ r \le z &\le 1 \end{aligned}$$

$$I = \int_0^{2\pi} d\theta \int_0^1 dr \int_r^1 r\cdot r\, dz = 2\pi \int_0^1 r^2\cdot(1-r)\, dr =$$

$$= 2\pi\left[\frac{r^3}{3} - \frac{r^4}{4}\right]\Big|_0^1 = 2\pi\left(\frac{1}{3}-\frac{1}{4}\right) = \frac{\pi}{6}$$

$$I = \iiint_T x^2\, dx\,dy\,dz \quad \text{.ב}$$

T חסום ע"י הספרה $x^2+y^2+z^2=9$



נעבור לקואורדינטות כדוריות

$$\begin{cases} x = \rho\cos\theta\sin\varphi \\ y = \rho\sin\theta\sin\varphi \\ z = \rho\cos\varphi \end{cases}$$

$0 \le \theta \le 2\pi$

$0 \le \varphi \le \pi$ $\qquad dx\,dy\,dz = \rho^2 \cdot \sin\varphi\, d\theta\, d\varphi\, d\rho$

$0 \le \rho \le 3$

$$I = \int_0^{2\pi} d\theta \int_0^{\pi} d\varphi \int_0^3 \rho^2\cos^2\theta\cdot\sin^2\varphi\cdot\rho^2\sin\varphi\, d\rho =$$

$$= \int_0^{2\pi} d\theta \int_0^{\pi} \cos^2\theta\cdot\sin^3\varphi\cdot\frac{\rho^5}{5}\Big|_{\rho=0}^{\rho=3} d\varphi =$$

$$= \int_0^{2\pi} d\theta \int_0^{\pi} \frac{3^5}{5}\cos^2\theta\cdot\sin^3\varphi\, d\varphi = \int_0^{2\pi}\frac{3^5}{5}\cos^2\theta\, d\theta\cdot$$

$$\cdot\int_0^{\pi}\sin^3\varphi\, d\varphi$$

$$\int_0^{2\pi}\cos^2\theta\, d\theta = \int_0^{2\pi}\frac{1+\cos 2\theta}{2}\, d\theta = \frac{1}{2}\cdot\left[\theta + \frac{1}{2}\sin 2\theta\right]\Big|_0^{2\pi} =$$

$$= \frac{1}{2}\cdot 2\pi = \pi$$

$$\int_0^{\pi}\sin^3\varphi\, d\varphi = \left\{\begin{matrix} t = \cos\varphi \\ dt = -\sin\varphi\, d\varphi \end{matrix}, \quad \begin{matrix} \sin^2\varphi = 1-\cos^2\varphi = \\ = 1-t^2 \end{matrix}\right\} =$$

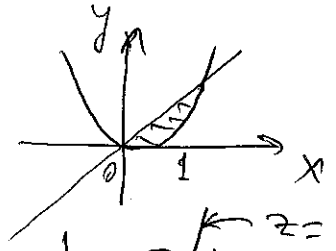
$$= -\int_1^{-1}(1-t^2)\, dt = \int_{-1}^{1}(1-t^2)\, dt = \left[t - \frac{t^3}{3}\right]\Big|_{-1}^{1} =$$

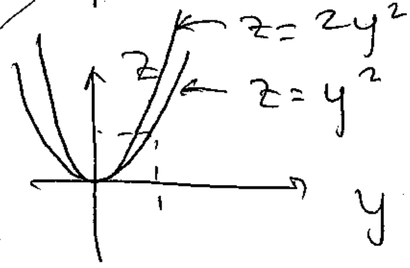
$$= 1 - \frac{1}{3} - \left(-1 + \frac{1}{3}\right) = \frac{4}{3}$$

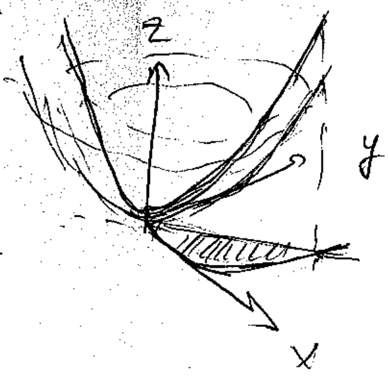
$$I = \frac{3^5}{5}\cdot\frac{4}{3}\cdot\pi = \frac{3^4\, 4}{5}\pi = \frac{324}{5}\pi$$

$z = x^2 + y^2 \quad z = 2x^2 + 2y^2$ ק. ③

$y = x \quad y = x^2$







$$V = \iint_D dx\,dy \int_{x^2+y^2}^{2x^2+2y^2} dz = \iint_D (x^2+y^2)\,dx\,dy =$$

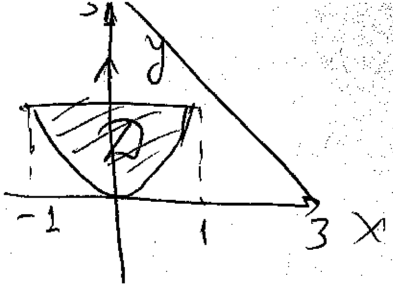
$$= \int_0^1 dx \int_{x^2}^{x} (x^2+y^2)\,dy = \int_0^1 \left[x^2 y + \frac{y^3}{3}\right]\Bigg|_{y=x^2}^{y=x} =$$

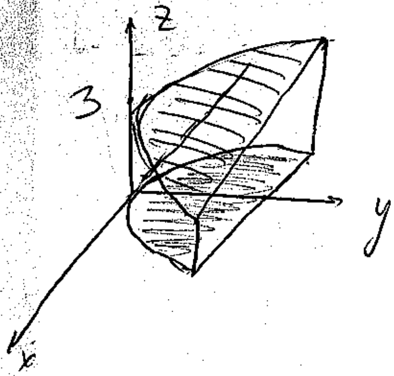
$$= \int_0^1 \left[x^3 + \frac{x^3}{3} - x^4 - \frac{x^6}{3}\right] dx = \left[\frac{4}{3}\cdot\frac{x^4}{4} - \frac{x^5}{5} - \right.$$

$$\left. - \frac{1}{3}\cdot\frac{x^7}{7}\right]\Bigg|_0^1 = \frac{1}{3} - \frac{1}{5} - \frac{1}{21} = \frac{6}{21} - \frac{1}{5} =$$

$$= \frac{2}{7} - \frac{1}{5} = \frac{10-7}{35} = \frac{3}{35}$$

$y = x^2, \quad y = 1, \quad x + y + z = 3, \quad z = 0$ ד.





$$V=\iint_D dx\,dy\int_0^{3-x-y} dz=\iint_D (3-x-y)\,dx\,dy=$$

$$=\int_{-1}^{1} dx\int_{x^2}^{1}(3-x-y)\,dy=\int_{-1}^{1}\left[(3-x)\cdot y-\frac{y^2}{2}\right]\Bigg|_{y=x^2}^{y=1}dx=$$

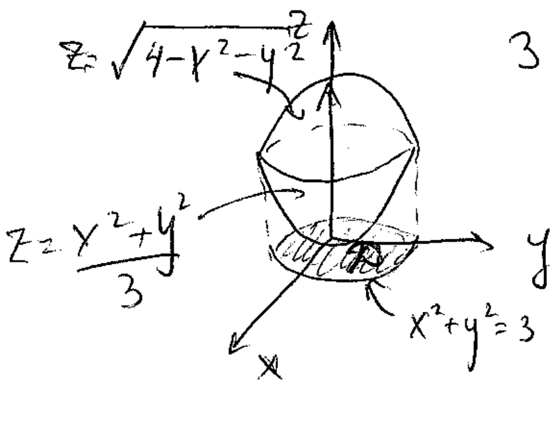
$$=\int_{-1}^{1}\left((3-x)\cdot(1-x^2)-\frac{1}{2}\cdot\left[1-x^4\right]\right)dx=$$

$$=\int_{-1}^{1}\left[3-3x^2-x+x^3-\frac{1}{2}+\frac{1}{2}x^4\right]dx=$$

$$=\left[\frac{5}{2}\cdot x-\frac{3x^3}{3}-\frac{x^2}{2}+\frac{x^4}{4}+\frac{1}{2}\frac{x^5}{5}\right]\Bigg|_{-1}^{1}=$$

$$=\frac{5}{2}-1-\frac{1}{2}+\frac{1}{4}+\frac{1}{10}-\left(-\frac{5}{2}+1-\frac{1}{2}+\frac{1}{4}-\frac{1}{10}\right)=$$

$$=5-2+\frac{2}{10}=3+\frac{1}{5}=\frac{16}{5}$$



$3z=x^2+y^2$, $z=\sqrt{4-x^2-y^2}$. d

$$z=\sqrt{4-x^2-y^2}\Rightarrow x^2+y^2+z^2=4$$

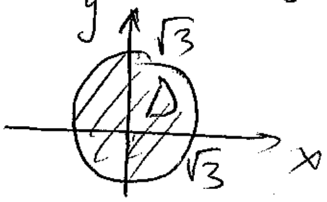
$$V=\iiint_T dx\,dy\,dz$$

$$\begin{cases} x^2+y^2+z^2=4 \\ x^2+y^2=3z \end{cases} \Rightarrow z^2+3z-4=0$$

$$z_{1,2}=-4,\ 1$$

$$z=1 \quad \Leftarrow \quad z>0$$

$$D:\ x^2+y^2\le 3$$



נעבור לקואורדינטות גליליות:

$$\begin{cases} x=r\cos\theta \\ y=r\sin\theta \\ z=z \end{cases}$$

$$dx\,dy\,dz=r\,d\theta\,dr\,dz$$

$$0\le\theta\le 2\pi$$

$$0\le r\le\sqrt{3}$$

$$\frac{r^2}{3}\le z\le\sqrt{4-r^2}$$

$$V=\int_0^{2\pi}d\theta\int_0^{\sqrt{3}}dr\int_{\frac{r^2}{3}}^{\sqrt{4-r^2}}r\,dz=$$

$$=2\pi\cdot\int_0^{\sqrt{3}}r\cdot\left(\sqrt{4-r^2}-\frac{r^2}{3}\right)dr=$$

$$=2\pi\left[\int_0^{\sqrt{3}}r\sqrt{4-r^2}\,dr-\int_0^{\sqrt{3}}\frac{r^3}{3}\,dr=\left(-\frac{1}{2}\,\frac{(4-r^2)^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_0^{\sqrt{3}}-\right.\right.$$

$$\left.-\frac{1}{3}\cdot\frac{r^4}{4}\Bigg|_0^{\sqrt{3}}\right)2\pi=\left[-\frac{1}{3}\cdot(1-8)-\frac{1}{3}\cdot\frac{9}{4}\right]2\pi=$$

$$=2\pi\cdot\left(\frac{7}{3}-\frac{3}{4}\right)=2\pi\cdot\left(\frac{28-9}{4\cdot 3}\right)=\frac{19}{6}\pi$$

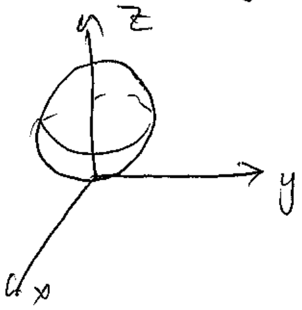
$$2z = x^2 + y^2 + z^2$$

$$x^2 + y^2 + (z-1)^2 = 1$$

$$V = \frac{4}{3}\pi \qquad \text{אלגנטי!}$$

$$\begin{cases} x = \rho\cos\theta \cdot \sin\varphi \\ y = \rho\sin\theta \cdot \sin\varphi \\ z = \rho\cos\varphi \end{cases}$$

$$dx\,dy\,dz = \rho^2 \sin\varphi \, d\rho\, d\theta\, d\varphi$$



$$x^2 + y^2 + z^2 = 2z$$

$$\Updownarrow$$

$$\rho^2 = 2\rho\cos\varphi$$

$$\rho = 2\cos\varphi$$

$$0 \le \theta \le 2\pi$$

$$0 \le \varphi \le \frac{\pi}{2}$$

$$0 \le \rho \le 2\cos\varphi$$

$$V = \int_0^{2\pi} d\theta \int_0^{\frac{\pi}{2}} d\varphi \int_0^{2\cos\varphi} \rho^2 \cdot \sin\varphi \, d\rho =$$

$$= 2\pi \int_0^{\frac{\pi}{2}} \sin\varphi \cdot \frac{\rho^3}{3}\Big|_0^{2\cos\varphi} d\varphi =$$

$$= \frac{2}{3}\pi \int_0^{\frac{\pi}{2}} 8\sin\varphi \cdot \cos^3\varphi \, d\varphi = \frac{16\pi}{3} \cdot \int_0^{\frac{\pi}{2}} \sin\varphi \cos^3\varphi \, d\varphi =$$

$$= \frac{\cancel{16}4}{3}\pi \cdot \left(-\frac{\cos^4\varphi}{4}\right)\Big|_0^{\frac{\pi}{2}} = \frac{4}{3}\pi$$

א. (4)

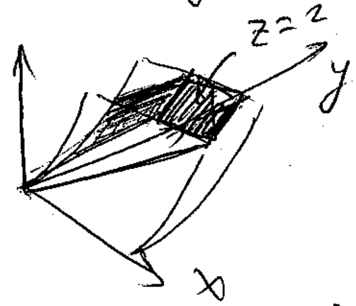
$$I=\iiint_T zy\cdot e^{x+y^2}\,dx\,dy\,dz$$

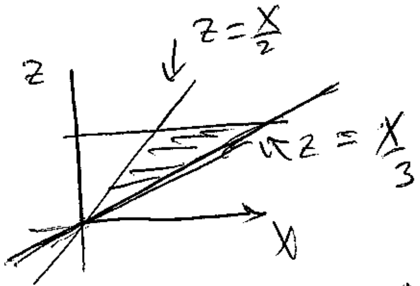
T: $0\le z\le 2$ כמובן $y\ge 0$

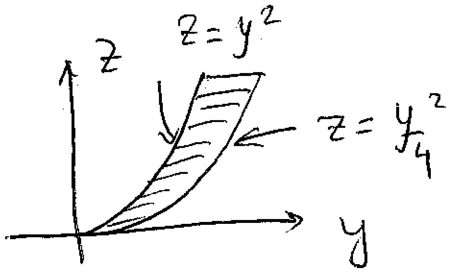
$\frac{x}{3}\le z\le \frac{x}{2}$

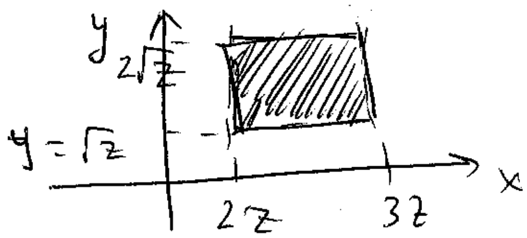
$\frac{y^2}{4}\le z\le y^2$

דרך 1









$$\iiint_T zy\cdot e^{x+y^2}dx\,dy\,dz=\int_0^2 dz\int_{2z}^{3z}dx\int_{\sqrt{z}}^{2\sqrt{z}} z\cdot y\cdot e^x e^{y^2}dy=$$

$$=\int_0^2 z\,dz\int_{2z}^{3z}e^x dz\int_{\sqrt{z}}^{2\sqrt{z}}y\cdot e^{y^2}dy=\int_0^2 z\,dz\int_{2z}^{3z}e^x\cdot\frac{1}{2}e^{y^2}\Big|_{y=\sqrt{z}}^{y=2\sqrt{z}}dz=$$

$$=\int_0^2 z\,dz\int_{2z}^{3z}e^x\cdot\frac{1}{2}\left(e^{4z}-e^{z}\right)dx=\int_0^2\frac{z}{2}\left(e^{4z}-e^{z}\right)\cdot$$

$$\left(e^{3z}-e^{2z}\right)dz=\int_0^2\frac{z}{2}\left(e^{7z}-e^{6z}-e^{4z}+e^{3z}\right)dz$$

$$\int z\cdot e^{az}dz = \left\{\begin{matrix} u=z & du=dz \\ dv=e^{az}dz & v=\frac{1}{a}e^{az}\end{matrix}\right\} =$$

$$= \frac{z\cdot e^{az}}{a} - \frac{1}{a}\int e^{az}dz = \frac{1}{a}\cdot z\cdot e^{az} - \frac{1}{a^2}e^{az} + C$$

$$I = \frac{1}{2}\int_0^2 \left(z\cdot e^{7z} - z\cdot e^{6z} - z\cdot e^{4z} + z\cdot e^{3z}\right)dz =$$

$$= \frac{1}{2}\cdot\Big[\frac{1}{7}\cdot z\cdot e^{7z} - \frac{1}{49}e^{7z} - \frac{1}{6}z\cdot e^{6z} + \frac{1}{36}e^{6z} -$$

$$- \frac{1}{4}ze^{4z} + \frac{1}{16}e^{4z} + \frac{1}{3}z\cdot e^{3z} - \frac{1}{9}e^{3z}\Big]\Big|_0^2 =$$

$$= \frac{1}{2}\cdot\Big[\frac{2}{7}\cdot e^{14} - \frac{1}{49}\cdot e^{14} - \frac{2}{6}\cdot e^{12} + \frac{1}{36}\cdot e^{12} -$$

$$- \frac{2}{4}\cdot e^{8} + \frac{1}{16}\cdot e^{8} + \frac{2}{3}\cdot e^{3\cdot 2} - \frac{1}{9}e^{6} -$$

$$- \left(-\frac{1}{49} + \frac{1}{36} + \frac{1}{16} - \frac{1}{9}\right)\Big]$$

<u>חלק 2</u> נעבור לקואורדינטות

$$\begin{cases} u = z \\ v = \dfrac{z}{x} \\ w = \dfrac{z}{y^2}, \quad y>0 \end{cases}$$

הגבלות הם מתאים ל $y>0$

$$0 \le u \le 2$$
$$\frac{1}{3} \le v \le \frac{1}{2}$$
$$\frac{1}{4} \le w \le 1$$

$$J^* = \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} 0 & 0 & 1 \\ -\frac{z}{x^2} & 0 & \frac{1}{x} \\ 0 & -\frac{2z}{y^3} & \frac{1}{y^2} \end{vmatrix} =$$

$$= \frac{z}{x^2} \cdot \frac{2z}{y^3}$$

$$|J| = \frac{1}{|J^*|} = \frac{x^2 \cdot y^3}{2z^2}$$: בתחום $y>0$

$$\frac{x^2 \cdot y^3}{2z^2} \cdot z \cdot y \cdot e^{x+y^2} = \frac{x^2 y^4}{2z} e^{x+y^2} =$$

$$= \frac{u^2}{v^2} \cdot \frac{u^2}{w^2} \cdot \frac{1}{2u} \cdot e^{\frac{u}{v}} \cdot e^{\frac{u}{w}}$$

$$I = \frac{1}{2}\int_0^2 du \int_{\frac{1}{3}}^{\frac{1}{2}} dv \int_{\frac{1}{4}}^{1} \frac{u^3}{v^2} e^{\frac{u}{v}} \cdot \frac{1}{w^2} \cdot e^{\frac{u}{w}} \, dw =$$

$$= \frac{1}{2}\int_0^2 du \int_{\frac{1}{3}}^{\frac{1}{2}} \frac{u^{3^2}}{v^2} \cdot \left(-\frac{1}{u}\right) \cdot e^{\frac{u}{v}} \cdot e^{\frac{u}{w}} \Big|_{w=\frac{1}{4}}^{w=1} dv =$$

$$= \frac{1}{2}\int_0^2 du \int_{\frac{1}{3}}^{\frac{1}{2}} \left(-u^2 (e^{u} - e^{4u}) \cdot e^{\frac{u}{v}} \cdot \frac{1}{v^2}\right) dv =$$

$$= \frac{1}{2}\int_0^2 \left[(e^{u} - e^{4u}) \cdot u^2 \cdot \frac{1}{u} \cdot e^{\frac{u}{v}} \Big|_{v=\frac{1}{3}}^{v=\frac{1}{2}} \right] du =$$

$$= \frac{1}{2}\int_0^2 u \cdot (e^{u} - e^{4u}) \cdot (e^{2u} - e^{3u}) \, du =$$

$$= \frac{1}{2}\int_0^2 u \cdot (e^{3u} - e^{4u} - e^{6u} + e^{7u}) \, du$$

פרק 3

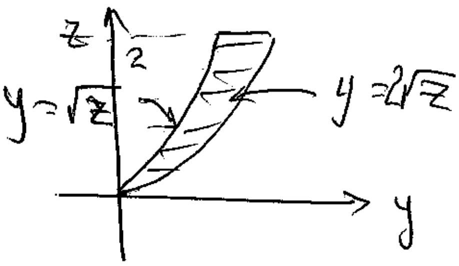
$$I=\iint\limits_{D_{zy}} dy\,dz \int_{2z}^{3z} z\cdot y\cdot e^{x+y^2}\,dx =$$

$$=\iint\limits_{D_{zy}} \left[z\cdot y\cdot e^{y^2}\cdot e^{x}\Big|_{x=2z}^{x=3z} \right] dy\,dz =$$

$$=\iint\limits_{D_{zy}} z\cdot y\cdot e^{y^2}\cdot\left(e^{3z}-e^{2z}\right) dy\,dz =$$

$$=\int_0^2 dz \int_{\sqrt{z}}^{2\sqrt{z}} z\left(e^{3z}-e^{2z}\right)\cdot y\cdot e^{y^2}\,dy =$$

$$=\int_0^2 z\cdot\left(e^{3z}-e^{2z}\right)\cdot \frac{1}{2}e^{y^2}\Big|_{y=\sqrt{z}}^{y=2\sqrt{z}}\,dz =$$



$$=\frac{1}{2}\int_0^2 z\cdot\left(e^{3z}-e^{2z}\right)\cdot\left(e^{4z}-e^{z}\right)dz =$$

$$=\frac{1}{2}\int_0^2 z\cdot\left(e^{7z}-e^{4z}-e^{6z}+e^{3z}\right)dz$$

ג. $I=\iiint\limits_{T}(x^2-y^2)\,dx\,dy\,dz$

$T:\ x-1\le y\le x$

$1-y\le x\le 2-y$

$y^2-x^2+1\le z\le y^2-x^2+2$

נעבור לקואורדינטות

$$\begin{cases} u=y-x\\ v=y+x\\ w=z-y^2+x^2\end{cases}$$

$$-1 \le u \le 0$$
$$1 \le v \le 2$$
$$1 \le w \le 2$$

$$J^* = \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 2x & -2y & 1 \end{vmatrix} = -2$$

$$|J| = \frac{1}{2}$$

$$+\begin{cases} u = y - x \\ v = y + x \end{cases} \qquad - \begin{array}{l} u = y - x \\ \underline{v = y + x} \end{array}$$
$$u + v = 2y \qquad u - v = -2x$$

$$\begin{cases} x = \frac{v-u}{2} \\ y = \frac{v+u}{2} \end{cases} \qquad x^2 - y^2 = \frac{1}{4}\cdot\left((v-u)^2 - (v+u)^2\right) =$$
$$= \frac{1}{4}\cdot(v-u-v-u)\cdot(v-u+v+u) =$$
$$= \frac{1}{4}(-2u)\cdot 2v = -u\cdot v$$

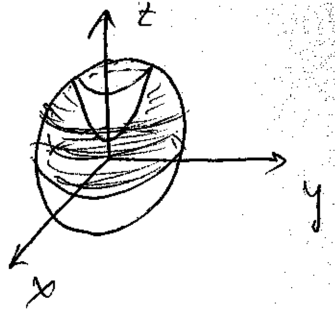
$$I = \frac{1}{2}\int_{-1}^{0} du \int_{1}^{2} dv \int_{1}^{2} (-uv)\, dw =$$

$$= \frac{1}{2}\int_{-1}^{0} du \int_{1}^{2} (-u\cdot v)\, dv = -\frac{1}{2}\int_{-1}^{0} u\cdot \frac{v^2}{2}\Big|_{v=1}^{v=2} du =$$

$$= -\frac{1}{2}\cdot\int_{-1}^{0} \frac{3}{2} u\, du = -\frac{3}{4}\int_{-1}^{0} u\, du = -\frac{3}{4}\cdot\frac{u^2}{2}\Big|_{-1}^{0} =$$

$$= \frac{3}{8}$$

5. א.



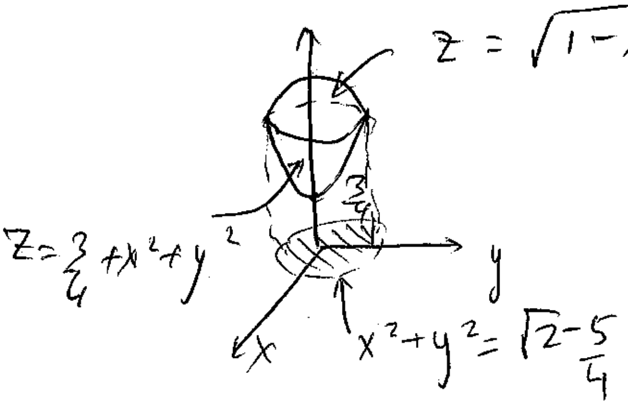
$V: \quad x^2+y^2+z^2 \le 1$

$0 \le z \le x^2+y^2+\frac{3}{4}$

$$V = \underset{\text{חצי כדור}}{V} - \underset{\text{נפח של גוף שבתוך פרבולויד}}{V_1}$$

$$\underset{\text{חצי כדור}}{V} = \frac{1}{2}\cdot\frac{4}{3}\pi\cdot 1^3 = \frac{2}{3}\pi$$

חישוב V_1



$$\begin{cases} x^2+y^2 = z-\frac{3}{4} \\ x^2+y^2+z^2 = 1 \end{cases}$$

$$z^2+z-\frac{3}{4}-1=0$$

$$z^2+z-\frac{7}{4}=0$$

$$z_{1,2} = \frac{-1\pm\sqrt{8}}{2};$$

$$z = \frac{\sqrt{8}-1}{2} \quad \Leftarrow z \ge 0$$

$$x^2+y^2 = \frac{\sqrt{8}-1}{2}-\frac{3}{4} = \frac{2\sqrt{8}-2-3}{4} = \frac{2\sqrt{8}-5}{4} =$$

$$= \sqrt{2}-\frac{5}{4}$$

נעבור לקואורדינטות גליליות:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

$$dx\,dy\,dz = r\,d\theta\,dr\,dz$$

$$0 \le \theta \le 2\pi$$

$$0 \le r \le \sqrt{\sqrt{2} - \frac{5}{4}} = r_0$$

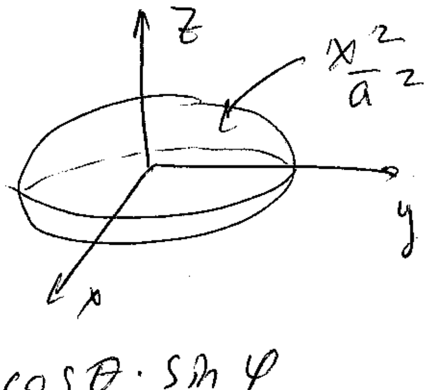
$$r^2 + \frac{3}{4} \le z \le \sqrt{1-r^2}$$

$$V_1 = \int_0^{2\pi} d\theta \int_0^{r_0} dr \int_{r^2+\frac{3}{4}}^{\sqrt{1-r^2}} r\,dz = 2\pi \cdot \int_0^{r_0} r\cdot\left(\sqrt{1-r^2} - r^2 - \frac{3}{4}\right) dr =$$

$$= 2\pi\left(- \frac{(1-r^2)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_0^{r_0} - \frac{r^4}{4}\Big|_0^{r_0} - \frac{3}{4}\frac{r^2}{2}\Big|_0^{r_0}\right) =$$

$$= 2\pi \cdot \left(\frac{2}{3} - \frac{2}{3}\sqrt{(1-r_0^2)^3} - \frac{r_0^4}{4} - \frac{3}{8} r_0 \right)$$

$$V = \frac{2}{3}\pi - 2\pi \cdot \left(\frac{2}{3} - \frac{2}{3}\sqrt{(1-r_0)^3} - \frac{r_0^4}{4} - \frac{3}{8} r_0 \right)$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

$$\begin{cases} x = a\rho\cos\theta\cdot\sin\varphi \\ y = b\rho\cdot\sin\theta\cdot\sin\varphi \\ z = c\cdot\rho\cos\varphi \end{cases}$$

$$0 \le \theta \le 2\pi$$
$$0 \le \varphi \le \pi$$
$$0 \le \rho \le 1$$

$$|J| = abc\rho^2 \sin\varphi$$

$$V = \int_0^{2\pi} d\theta \int_0^{\pi} d\varphi \int_0^1 abc\,\rho^2 \cdot \sin\varphi\, d\rho = 2\pi \int_0^{\pi} \frac{1}{3} abc \sin\varphi\, d\varphi =$$

$$= \frac{2\pi}{3} abc \left(-\cos\varphi\right)\Big|_0^{\pi} = \frac{4\pi}{3} abc$$

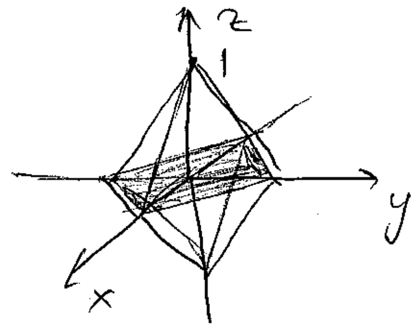
ד:

$$|x+2y+3z| + |2x+3y+z| + |3x+y+2z| \le 1$$

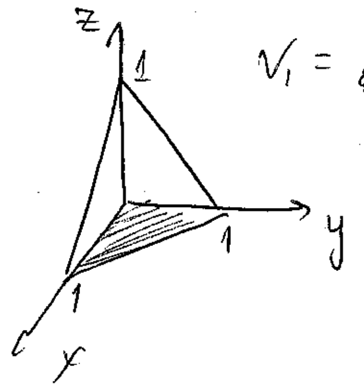
נעבור לקואורדינטות

$$\begin{cases} u = x+2y+3z \\ v = 2x+3y+z \\ w = 3x+y+2z \end{cases}$$

$$|u| + |v| + |w| \le 1$$



$$V = 8 \cdot V_1$$



$$V_1 = \cancel{8}\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

$$J^* = \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} =$$

$$= 6-1-2\cdot(4-3)+3\cdot(2-9) =$$

$$= 5-2-21 = 3-21 = -18$$

$$|J| = \frac{1}{18}$$

$$V = \overset{4}{\cancel{8}} \cdot \frac{1}{\underset{9}{\cancel{18}}} \cdot \frac{1}{6} = \frac{4}{9\cdot 6} = \frac{2}{9\cdot 3} = \frac{2}{27}$$

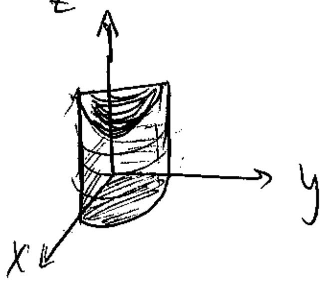
(6)

$$I=\iiint_T (x^2+y^2)\,dx\,dy\,dz$$

T : $x^2+y^2=4$ $\quad x\geq 0$

$z=1+x^2+y^2$ $\quad y\geq 0$

$z\geq 0$



נעבור לקואורדינטות

גליליות

$$\begin{cases} x=r\cos\theta \\ y=r\sin\theta \\ z=z \end{cases} \qquad dx\,dy\,dz = r\,dr\,d\theta\,dz$$

$$\begin{cases} 0\leq\theta\leq\frac{\pi}{2} \\ 0\leq r\leq 2 \\ 0\leq z\leq 1+r^2 \end{cases}$$

$$I=\int_0^{\frac{\pi}{2}}d\theta\int_0^2 dr\int_0^{1+r^2} r\cdot(r^2)\,dz=$$

$$=\frac{\pi}{2}\int_0^2 r^3\cdot(1+r^2)\,dr=\frac{\pi}{2}\cdot\left(\frac{r^4}{4}+\frac{r^6}{6}\right)\Bigg|_0^2=$$

$$=\frac{\pi}{2}\cdot\left(\frac{16}{4}+\frac{32\cdot 2}{6}\right)=\frac{\pi}{2}\cdot\left(4+\frac{32}{3}\right)=\cancel{\frac{\pi\cdot 52}{2\cdot 5}=\frac{26}{5}\pi}$$

$$=\pi\cdot\left(2+\frac{16}{3}\right)=\frac{22}{3}\pi$$

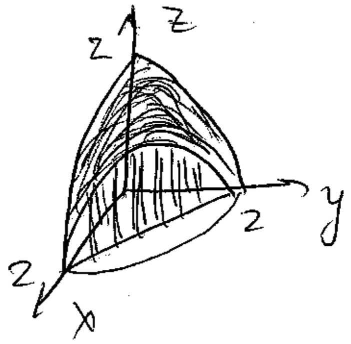
7

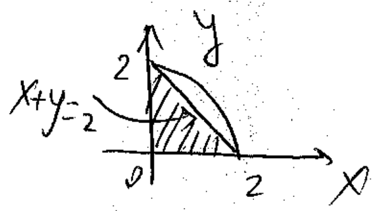
$$I = \iiint_T z \, dx\, dy\, dz$$

$$x^2 + y^2 + z^2 \le 4$$

$$x + y \le 2$$

$$x \ge 0, \quad y \ge 0, \quad z \ge 0$$





$$I = \int_0^2 dx \int_0^{2-x} dy \int_0^{\sqrt{4-x^2-y^2}} z \, dz =$$

$$= \frac{1}{2} \int_0^2 dx \int_0^{2-x} (4 - x^2 - y^2)\, dy =$$

$$= \frac{1}{2} \int_0^2 \left[(4-x^2)\cdot y - \frac{y^3}{3} \right] \Big|_{y=0}^{y=2-x} dx =$$

$$= \frac{1}{2} \int_0^2 \left[(4-x^2)\cdot(2-x) - \frac{1}{3}(2-x)^3 \right] dx =$$

$$= \frac{1}{2} \cdot \left(8x - \frac{4x^2}{2} - 2\frac{x^3}{3} + \frac{x^4}{4} + \frac{1}{3} \cdot \frac{(2-x)^4}{4} \right) \Big|_0^2 =$$

$$= \frac{8}{3}$$