Exercise 6—Calculus 2 (Spring 2016)

Ex1. At what point on the paraboloid $y = x^2 + z^2$ is the tangent plane parallel to the plane $x + 2y + 3z = 1$?

Ex2. Find the linearization of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $P(1, 0, 0)$.

Ex3. Evaluate $\frac{dw}{dt}(1)$ where $w = 2ye^x - \ln z$, $x = \ln(t^2 + 1)$, $y = \tan^{-1} t$ and $z = e^t$.

Ex4. Evaluate $\frac{dw}{du}(u, v)$ and $\frac{dw}{dv}(u, v)$ where $w = xy + yz + xz$, $x = u + v$, $y = u - v$ and $z = uv$ at the point $(u, v) = (\frac{1}{2}, 1)$.

Ex5. Let $T(x, y) = 4x^2 - 4xy + 4y^2$ be the temperature at the point (x, y). Find minimum and maximum values of $T(x, y)$ on the circle $x = \cos t, y = \sin t$ where $0 \leq t < 2\pi$.

Ex6. Find $D_u f(1, 1, 1)$ where $f(x, y, z) = x^2 + 2y^2 - 3z^2$ and $u = \frac{1}{\sqrt{3}}(1, 1, 1)$.

Ex7. Find the tangent plane and the normal line to the surface $x^2 - xy - y^2 - z = 0$ at the point $P(1, 1, -1)$.

Ex8. Find the equation of the tangent line to the curve of intersection of surfaces $xyz = 1$ and $x^2 + 2y^2 + 3z^2 = 6$ at the point $P(1, 1, 1)$.

Ex9. Find and classify all extremal points of the function $f(x, y) = x^3 + 3xy + y^3$.

Ex10. A function $f(x, y)$ is homogeneous of degree $n \geq 0$ if $f(tx, ty) = t^n f(x, y)$ for all t, x, y. For such function show that:
1. $x\frac{df}{dx} + y\frac{df}{dy} = nf(x, y)$
2. $x^2\frac{d^2f}{d^2x} + y^2\frac{d^2f}{d^2y} + 2xy\frac{d^2f}{dxdy} = n(n-1)f(x, y)$.

Ex11. Let $f(x, y)$ be a differentiable function at (x_0, y_0). Which of the following statements are true?
1. $D_u f(x_0, y_0) = f_x(x_0, y_0)\frac{u_1}{|u|} + f_y(x_0, y_0)\frac{u_2}{|u|}$, where $u = (u_1, u_2)$.

2. $D_u f(x_0, y_0)$ is a vector.
3. $D_u f(x_0, y_0)$ has its greatest value at $u = \nabla f(x_0, y_0)$.
4. At (x_0, y_0) the vector ∇f is normal to the curve $f(x, y) = f(x_0, y_0)$.

Ex12. Find two numbers $b > a$ such that $\int_a^b (24 - 2x - x^2)^{\frac{1}{3}} dx$ is maximal.

Ex13. Show that $(0, 0)$ is a critical point of the function $f(x, y) = x^2 + kxy + y^2$ for every k.

Ex14. For what values of k the second derivative test guaranties that the function $f(x, y) = x^2 + kxy + y^2$ has
1. a saddle point at $(0, 0)$
2. a minimum at $(0, 0)$
3. a maximum at $(0, 0)$
4. the test is inconclusive.

Ex15. Among all the points on the graph of $z = 10 - x^2 - y^2$ that lie above the plane $x + 2y + 3z = 0$, find the point farthest from the plane.

Ex16. Find the point on the graph of $z = x^2 + y^2 + 10$ nearest to the plane $x + 2y - z = 0$.