

Exercise 5—Calculus 2 (Spring 2016)

Ex1. Reparametrize the curve $r(t) = (e^{2t} \cos(2t), 2, e^{2t} \sin(2t))$ with respect to the arclength measured from $t = 0$ in the direction of increasing t . Find its length when $0 \leq t \leq 2$.

Ex2. Find and sketch the domain of the function $f(x, y) = \frac{\sqrt{y-x^2}}{1-x^2}$.

Ex3. Sketch the level curve $f(x, y) = c$ of the function $f(x, y) = \sum_{n=0}^{\infty} (\frac{x}{y})^n$ that passes through the point $(1, 2)$.

Ex4. Find the domain and the range of the function $f(x, y) = x^2 - y^2$. Sketch the level curves $f(x, y) = c$ for $c = -1, 0, 1$. Sketch the graph $z = f(x, y)$.

Ex5. Find the limit if it exists, or show that it does not exist.

a. $\lim_{(x,y) \rightarrow (0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}$

b. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 \sin^2 y}{x^2 + 3y^2}$

c. $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$

d. $\lim_{(x,y) \rightarrow (0,0)} \frac{e^{-(x^2+y^2)} - 1}{x^2 + y^2}$

Ex6. Let $f(x, y, z) = \sqrt{1 + xz} + \sqrt{1 - xy}$. Find f_{xyz} .

Ex7. Let $f(x, y) = \frac{2xy}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Compute $f_x(0, 0)$ and $f_y(0, 0)$, and show that f_x and f_y are not continuous at $(0, 0)$. Is it true that $f(x, y)$ is differentiable?

Ex8. Let $f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Determine where f_x and f_y exist and where are they continuous? Compute f_{xy} and f_{yx} at $(0, 0)$.