Exercise 4— (Spring 2016)

Ex1. Show that the distance from a point $P(x_1, y_1)$ to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Ex2. Find an equation of the plane determined by the lines $L_1(t) = (2t+1, 3t+2, 4t+3)$ and $L_2(s) = (s + 2, 2s + 4, -4s - 1)$.

Ex3. Show that the distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is

$$\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

Ex4. Let L_1 be the line through the points $(1, 2, 6)$ and $(2, 4, 8)$. Let L_2 be the line of intersection of two planes, where the first plane is given by equation $x - y + 2z + 1 = 0$ and the second plane passes through $(3, 2, -1)$, $(0, 0, 1)$ and $(1, 2, 1)$. Find the distance between L_1 and L_2.

Ex5. Let $u(t), v(t), w(t)$ differentiable functions. Show that

$$[u(t) \cdot (v(t) \times w(t))]' = u'(t) \cdot (v(t) \times w(t)) + u(t) \cdot (v'(t) \times w(t)) + u(t) \cdot (v(t) \times w'(t))$$

Ex6. Let $u(t) = r(t) \cdot (r'(t) \times r''(t))$. Show that $u'(t) = r(t) \cdot (r'(t) \times r'''(t))$.

Ex7. Find the length of the curve $r(t) = (t \sin t + \cos t, t \cos t - \sin t)$ when $\sqrt{2} \leq t \leq 2$ and find an equation of the tangent line that passes through the point $r(\sqrt{2})$.

Ex8. Find the arclength parameter $s(t) = \int_0^t |r'(\tau)| d\tau$ of the curve $r(t) = (e^t \cos t, e^t \sin t, e^t)$ and the length of the curve $r(t)$ when $-\ln 4 \leq t \leq 0$.